Complete Axiomatisation of Modal μ-Calculus

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LFCS

Modal μ-Calculus

- Modal Logic + (Least) Fixpoint Operator
- Example:

$$p \wedge \langle a \rangle p$$

$$\mu Z . p \vee \langle a \rangle p \qquad \text{eventually p}$$

$$\mu Y . \nu Z . (p \wedge [a] Y) \vee (\neg p \wedge [a] Z) \qquad \text{p finitely often}$$

- Very expressive logic for describing properties of processes

Kozen's Axiomatisation

- AX = Axioms for Modal logic (K) +

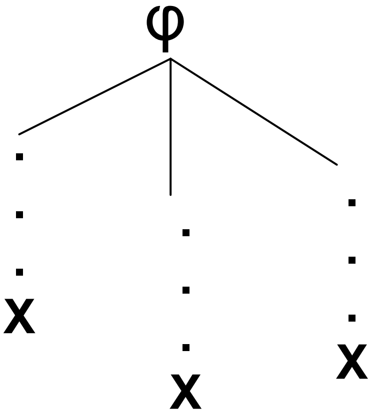
  $\cdot\, \Phi(\mu Z.\Phi(Z)) \rightarrow \mu Z.\Phi(Z)$

  $\cdot\, \text{if } \Phi(\Psi) \rightarrow \Psi \text{ then } (\mu Z.\Phi(Z)) \rightarrow \Psi$

- **Problem:** AX is sound and complete.

- Kozen(1983) proved completeness for "aconjunctive" fragment.
- Walukiewicz (1995) published a highly involved proof for full logic.
- **Goal:** Find an alternative and/or simplified completeness proof.

Tableaux



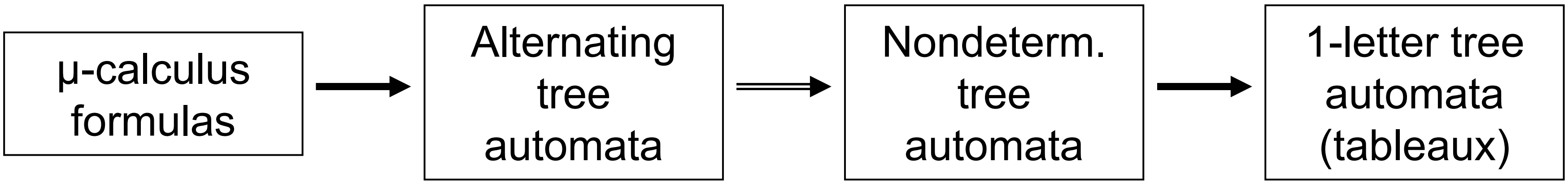
$$\longleftrightarrow \quad \vdash \neg\varphi$$

- φ has a closed tableau iff ¬φ is provable.
- No tableau system (that is finite and simple enough) found.

Games

- Lange and Stirling's satisfiability checking game for LTL, CTL and aconjunctive μ-calculus.
- Player 1 has a winning strategy for G(φ) iff φ is satisfiable.
- Easily yields completeness axiomatisation.
- Still not known how to extend to full logic.

Automata



$$M(\varphi) \quad = \quad L(A_{ata}) \quad = \quad L(A_n)$$

$$L(A_n) = \phi \quad \leftrightarrow \quad L(A_n^1) = \phi$$

- Tableaux = 1-letter nondeterm tree automata.
- Conversion from alternating automata to nondeterm automata is complicated.
- Still not known how to prove completeness directly from alternating automata.